Credit Risk Simulation
Counterparty Credit Risk (CCR) Definition

- Counterparty credit risk refers to the risk that a counterparty in a financial contract could fail to honor its obligation that causes financial loss to the non-defaulting party.
- Counterparty risk is present in financial market, especially in over-the-counter (OTC) derivatives and financial security transactions (FSTs) (e.g., repos).
- If one party of a contract defaults, the non-defaulting party will find a similar contract with another counterparty in the market to replace the default one. That is why counterparty credit risk sometimes is referred to as replacement risk.
CCR Simulation

Monte Carlo Simulation

- To calculate credit exposure or replacement cost in future times, one needs to simulate market evolutions.
- Simulation must be conducted under the real-world measure.
- Simple solution
  - Some vendors and institutions use this simplified approach
  - Only a couple of stochastic processes are used to simulate all market risk factors.
  - Use Vasicek model for all mean reverting factors
    \[ dr = k(\theta - r)dt + \sigma dW \]
    where \( r \) - risk factor; \( k \) - drift; \( \theta \) - mean reversion parameter; \( \sigma \) - volatility; \( W \) - Wiener process.
Monte Carlo Simulation (Cont’d)

- Use Geometric Brownian Motion (GBM) for all non-mean reverting risk factors.

\[ dS = \mu S dt + \sigma S dW \]

where \( S \) – risk factor; \( \mu \) – drift; \( \sigma \) – volatility; \( W \) – Wiener process.

- Different risk factors have different calibration results.

Complex solution

- Different stochastic processes are used for different risk factors.
- These stochastic processes require different calibration processes.
- Discuss this approach in details below.
Yield curve simulation

- Simulate yield curves (zero rate curves) or swap curves.
- There are many points in a yield curve, e.g., 1d, 1w, 2w 1m, etc. One can use Principal Component Analysis (PCA) to reduce risk factors from 20 points, for instance, into 3 point drivers.
- Using PCA, you only need to simulate 3 drivers for each curve. But please remember you need to convert 3 drivers back to 20-point curve at each path and each time step.
One popular yield curve simulation model under the real-world measure is the Cox-Ingersoll-Ross (CIR) model.

\[ dr = k(\theta - r)dt + \sigma \sqrt{r} dW \]

where \( r \) – risk factor; \( k \) – drift; \( \theta \) – mean reversion parameter; \( \sigma \) – volatility; \( W \) – Wiener process.

Reasons for choosing the CIR model

- Generate positive interest rates.
- It is a mean reversion process: empirically interest rates display a mean reversion behavior.
- The standard derivation in short term is proportional to the rate change.
Simulate foreign exchange rates.

Black Karasinski (BK) model:

\[ d \ln(r) = k(\ln(\theta) - \ln(r))dt + \sigma dW \]

where \(r\) – risk factor; \(k\) – drift; \(\theta\) – mean reversion parameter; \(\sigma\) – volatility; \(W\) – Wiener process.

Reasons for choosing BK model:

- Lognormal distribution;
- Non-negative FX rates;
- Mean reversion process.
Simulate stock prices.

Geometric Brownian Motion (GBM)

\[ dS = \mu S dt + \sigma S dW \]

where \( S \) – stock price; \( \mu \) – drift; \( \sigma \) – volatility; \( W \) – Wiener process.

Pros
- Simple
- Non-negative stock price

Cons
- Simulated values could be extremely large for a longer horizon, so it may be better to incorporate with a reverting draft.
Commodity simulation

- Simulate commodity spot, future and forward prices, pipeline spreads and commodity implied volatilities.
- Two factor model

\[
\begin{align*}
\log(S_t) &= q_t + x_t + y_t \\
\frac{dx_t}{dt} &= (\alpha_1 - \gamma_1 x_t) dt + \sigma_1 dW^1_t \\
\frac{dy_t}{dt} &= (\alpha_2 - \gamma_2 y_t) dt + \sigma_2 dW^2_t \\
\text{d}W^1_t \text{d}W^2_t &= \rho \text{d}t
\end{align*}
\]

where \( S_t \) – spot price or spread or implied volatility; \( S_t \) – deterministic function; \( x_t \) – short term deviation and \( y_t \) – long term equilibrium level.
- This model leads to a closed form solution for forward prices and thereby forward term structures.
Simulate equity or FX implied volatility.

Empirically implied volatilities are more volatile than prices.

Stochastic volatility model, such as Heston model

\[
\begin{align*}
\frac{dS_t}{S_t} &= \mu dt + \sqrt{V_t} S_t dW_1^t \\
\frac{dV_t}{V_t} &= \kappa (\theta - V_t) dt + \xi \sqrt{V_t} dW_2^t \\
\end{align*}
\]

\[dW_1^t dW_2^t = \rho dt\]

Where \(S_t\) is the implied volatility and \(V_t\) is the instantaneous variance of the implied volatility.

Pros

- Simulated distribution has fat tail or large skew and kurtosis.
Implied volatility simulation (Cont’d)

- **Cons**
  - Complex implementation
  - Unstable calibration

- If a stochastic volatility model is too complex to use, a simple alternative is

\[ dr = k(\theta - r)dt + \sigma dW \]

where \( r \) - volatility risk factor; \( k \) - drift; \( \theta \) - mean reversion parameter; \( \sigma \) - volatility; \( W \) - Wiener process.
Thanks!

You can find more details at
https://finpricing.com/lib/IrCurveIntroduction.html